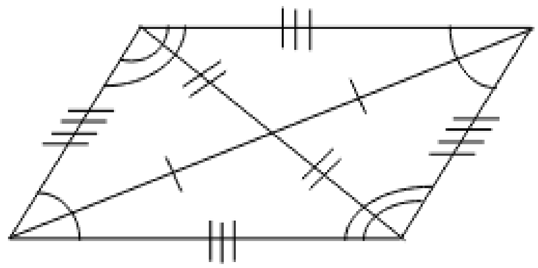
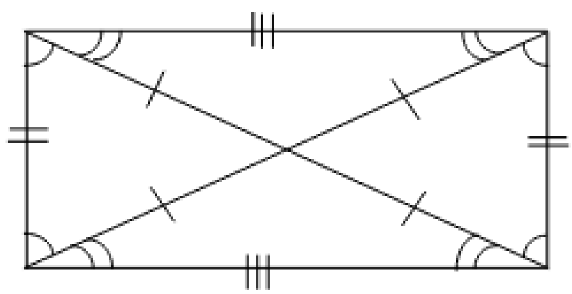
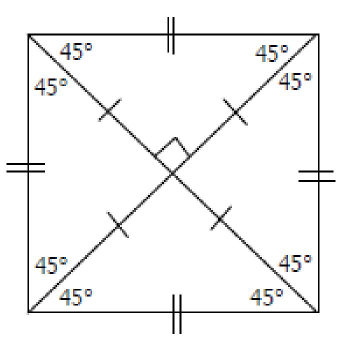
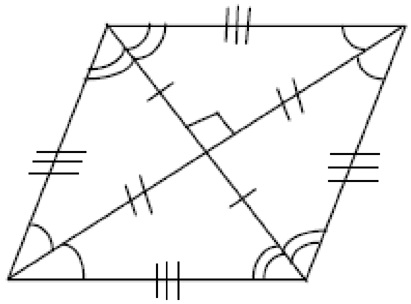
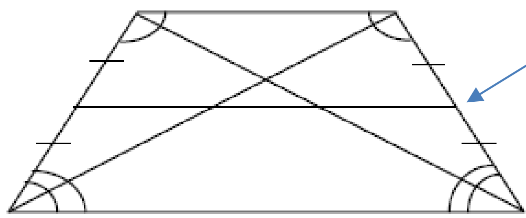
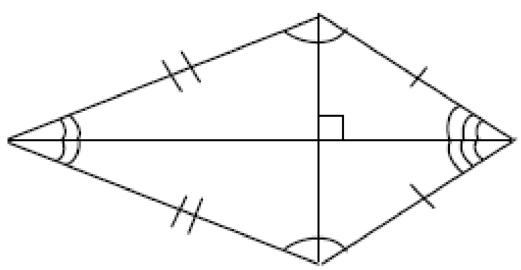
Geometry

- **Properties of Quadrilaterals**

Quadrilaterals are four-sided polygons with properties dependent on their side lengths, angles, and symmetry. Common quadrilaterals include squares, rectangles, parallelograms, rhombuses, and trapezoids. Each type has distinct properties. The chart below summarizes the important characteristics of quadrilaterals that display specific symmetries.

	PARALLELOGRAMS: (rectangles, squares, and rhombi) 1) Opposite sides of a parallelogram are congruent (they have the same length) and parallel. 2) Opposite angles of a parallelogram are congruent (they have the same measurement). 3) Consecutive angles in a parallelogram are supplementary. 4) The diagonals of a parallelogram bisect each other (they cut each other into equal parts).
	RECTANGLES: 1) Opposite sides of a rectangle are congruent and parallel. 2) Opposite angles are congruent. 3) Consecutive angles are supplementary. 4) The diagonals of a rectangle bisect each other. 5) The diagonals of a rectangle are congruent. 6) All four corner angles are 90°.
	SQUARES: 1) Opposite sides are congruent and parallel. 2) Opposite angles are congruent. 3) Consecutive angles are supplementary. 4) The diagonals of a square bisect each other, and they are perpendicular (they form a right angle). 5) The diagonals of a square are congruent. 6) The diagonals of a square bisect corner angles (they cut each corner angle into two equal halves). 7) All four corner angles are 90°. 8) All four sides are congruent.

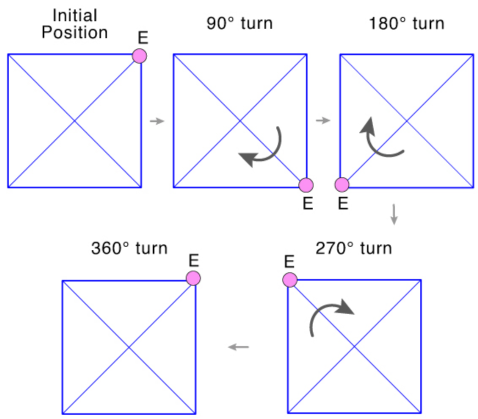
	<u>RHOMBI</u>: 1) Opposite sides are congruent and parallel. 2) Opposite angles are congruent. 3) Consecutive angles are supplementary. 4) The diagonals of a rhombus bisect each other, and they are perpendicular. 5) The diagonals of a square bisect corner angles. 6) All four sides are congruent.
	<u>ISOSCELES TRAPEZOIDS</u>: Midsegment $= \frac{1}{2} \cdot$ (upper base $+$ lower base) 1) The two bases are parallel. 2) Lower two base angles are congruent. 3) Upper two base angles are congruent. 4) The diagonals are congruent. 5) Two pairs of consecutive angles along the non-parallel sides (lateral sides) are supplementary. 6) Opposite angles are supplementary.
	<u>KITES</u>: 1) Two pairs of consecutive sides congruent, but opposite sides not congruent. 2) The diagonals of a kite are perpendicular. 3) The diagonals of a kite bisect one pair of corner angles. 4) Exactly one pair of angles congruent.

- **Rotational Symmetry**

Rotational symmetry in shapes refers to a shape's ability to look the same after some rotation by a certain angle less than a full 360 degrees. This concept is a fundamental aspect of geometry and is often used to analyze patterns and structures in both natural and human-made objects.

To determine if a shape has rotational symmetry, you rotate it around its *center point*. If the shape looks identical at least once before making a complete turn, it has rotational symmetry. The smallest angle at which this occurs is called the ***angle of rotation***. The number of times the shape matches its original position as it completes a full 360-degree rotation determines its ***order of rotational symmetry***.

For example, a *square* has rotational symmetry because it looks the same at 90° intervals ($360^\circ/4$). Therefore, it has an order of 4.



Rotational Symmetry

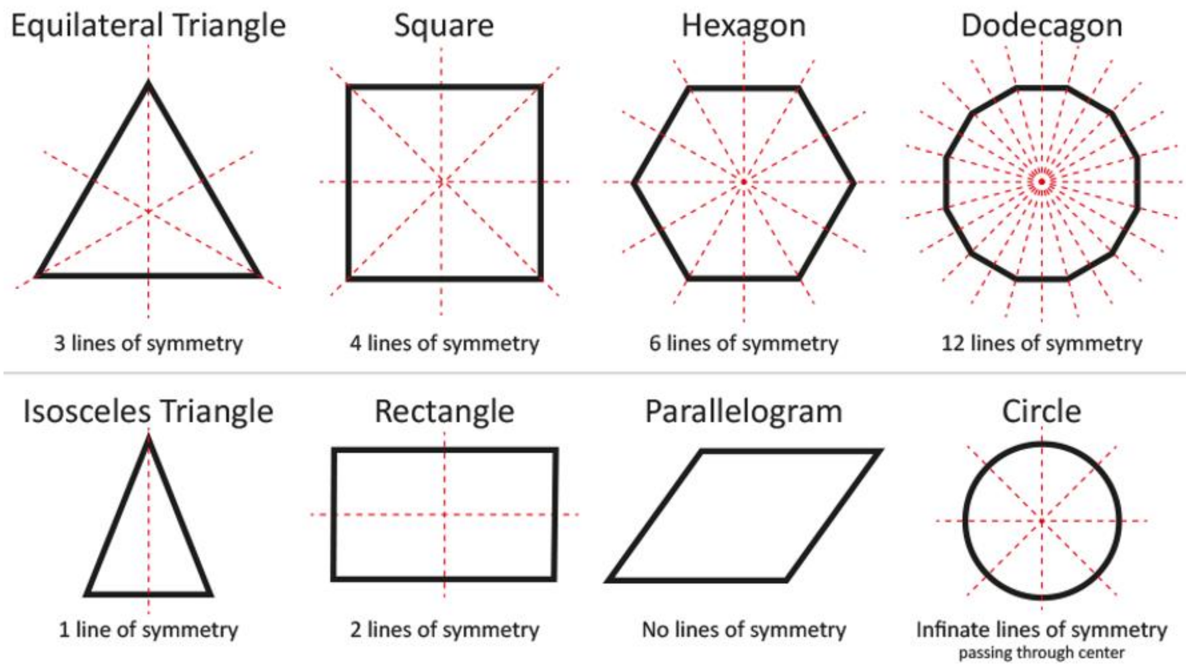
Other shapes, like rectangles, have an order of 2 because they only align with their original positions twice during a full rotation (at 0 and 180 degrees). Shapes that look the same at every angle, like a circle, have *infinite* rotational symmetry. Understanding rotational symmetry helps in fields ranging from art and design to mathematics and engineering.

Shape	**Order of Rotational Symmetry (x)**	**Angle of Rotational Symmetry ($360°/x$)**
Rectangle	2	180°
Equilateral Triangle	3	120°
Square	4	90°
Regular Pentagon	5	72°
Regular Hexagon	6	60°
Regular Heptagon	7	51.43°
Regular Octagon	8	45°
Regular Nonagon	9	40°
Regular Decagon	10	36°

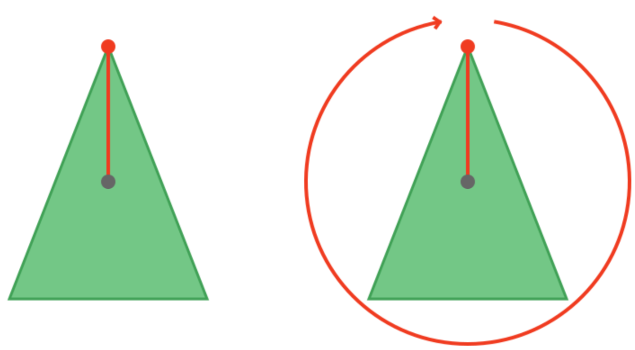
- **Lines of Symmetry**

In geometry, the lines of symmetry of a shape are imaginary lines that you can draw through the shape such that each side of the line is a mirror image of the other. Counting the lines of symmetry helps in understanding the balance and aesthetics of various shapes.

For regular polygons, the number of lines of symmetry is equal to the number of sides of the regular polygon.

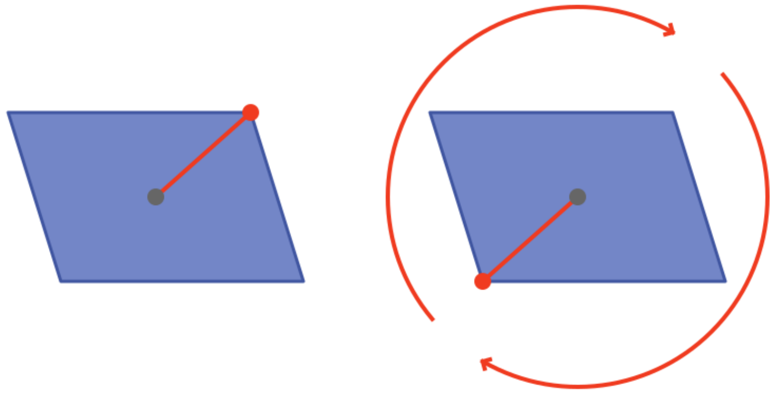


The concept of rotational symmetry and lines of symmetry are related but **not** equivalent. For example, an isosceles triangle has 1 line of symmetry as shown in the diagram above. However, an isosceles triangle has no rotational symmetry. A shape with no rotational symmetry has its order of rotational symmetry equal to 1 and its angle of rotational symmetry equal to 360°.

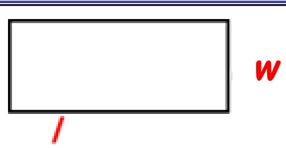
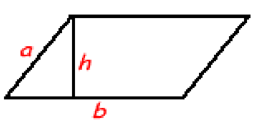
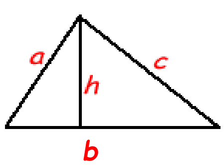
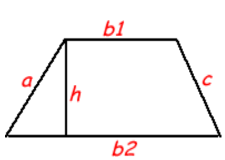


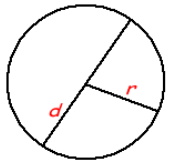
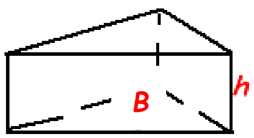
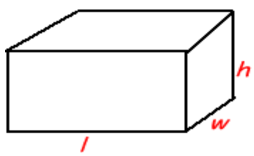
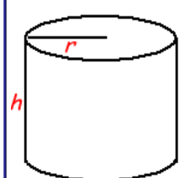
Example: Determine the order and angle of rotational symmetry of a parallelogram.

The order of rotational symmetry of a parallelogram is 2 and the angle of rotational symmetry is $180°$, as shown in the diagram below.



- **Geometric Formulae**

Shapes	**Formula** $\mathbf{2-dimensional: A = Area, P = Perimeter, C = Circumference,}$ $\mathbf{3-dimensional: V = Volume, SA = Surface\ Area}$
	**Rectangle:** $A = lw$ $P = 2l + 2w$
	**Parallelogram:** $A = bh$ $P = 2a + 2b$
	**Triangle:** $A = \frac{bh}{2}$ $P = a + b + c$
	**Trapezoid:** $A = \frac{(b_1 + b_2) \cdot h}{2}$ $P = a + b_1 + b_2 + c$

	**Circle:** The distance around a circle is the circumference C of the circle. The distance across a circle through its centre is the diameter d of the circle, and the radius r is the distance from the center to a point on the circle. $d = 2r, C = \pi d = 2\pi r, A = \pi r^2$
	**Prisms** $V = Bh$, where B is the area of the cross section (base) of the prism. $SA = 2B + Ph$, where P is the perimeter of the cross section (base) of the prism.
	**Rectangular Solid:** **(One type of Prisms)** $V = lwh$ $SA = 2lw + 2lh + 2wh = 2(lw + lh + wh)$
	**Cylinder:** **(One type of Prisms)** $V = \pi r^2 h$ $SA = 2\pi r^2 + 2\pi rh$