

Ratio, Rate, and Percent

1. Ratio

A ratio is a comparison of two numbers. We generally separate the two numbers in the ratio with a colon (:). Suppose we want to write the ratio of 8 and 12.

We can write this as 8:12 or as a fraction $8/12$, and we say the ratio is *eight to twelve*.

2. Comparing Ratios

To compare ratios, write them as fractions. The ratios are equal if they are equal when written as fractions.

Example:

Are the ratios 3 to 4 and 6:8 equal?

The ratios are equal if $3/4 = 6/8$.

These are equal if their cross products are equal; that is, if $3 \times 8 = 4 \times 6$. Since both of these products equal 24, the answer is yes, the ratios are equal.

3. Rate

A rate is a ratio that expresses how long it takes to do something, such as traveling a certain distance. To walk 3 kilometers in one hour is to walk at the rate of 3 km/h. The fraction expressing a rate has units of distance in the numerator and units of time in the denominator.

Problems involving rates typically involve setting two ratios equal to each other and solving for an unknown quantity, that is, solving a proportion.

Example:

Juan runs 4 km in 30 minutes. At that rate, how far could he run in 45 minutes?

Give the unknown quantity the name n . In this case, n is the number of km Juan could run in 45 minutes at the given rate. We know that running 4 km in 30 minutes is the same as running n km in 45 minutes; that is, the rates are the same. So we have the proportion $4\text{km}/30\text{min} = n\text{ km}/45\text{min}$, or $4/30 = n/45$.

Finding the cross products and setting them equal, we get $30 \times n = 4 \times 45$, or $30 \times n = 180$. Dividing both sides by 30, we find that $n = 180 \div 30 = 6$ and the answer is 6 km.

4. Proportion

A proportion is an equation with a ratio on each side. It is a statement that two ratios are equal. $\frac{3}{4} = \frac{6}{8}$ is an example of a proportion.

When one of the four numbers in a proportion is unknown, cross products may be used to find the unknown number. This is called solving the proportion. Question marks or letters are frequently used in place of the unknown number.

Example:

Solve for n : $\frac{1}{2} = \frac{n}{4}$.

Using cross products we see that $2 \times n = 1 \times 4 = 4$, so $2 \times n = 4$. Dividing both sides by 2, $n = 4 \div 2$ so that $n = 2$.

5. Percent

A percent is a ratio whose second term is 100. Percent means parts per hundred. The word percent comes from the Latin phrase *per centum*, which means per hundred. In mathematics, we use the symbol % for percent.

Let's look at our comparison table again. This time the table includes percents.

Comparing Shaded Boxes to Total Boxes			
Grid	Ratio	Fraction	Percent
1	96 to 100	$\frac{96}{100}$	96%
2	9 to 100	$\frac{9}{100}$	9%
3	77 to 100	$\frac{77}{100}$	77%

Example: Write each ratio as a fraction, a decimal, and a percent: 4 to 100, 63 to 100, 17 to 100

Solution			
Ratio	Fraction	Decimal	Percent
4 to 100	$\frac{4}{100}$.04	4%
63 to 100	$\frac{63}{100}$.63	63%
17 to 100	$\frac{17}{100}$.17	17%

Questions in class

1. After running 25% of a race, Giselle had run 50 metres. How long was the race, in metres?
2. The ratio of the number of big dogs to the number of small dogs at a pet show is 3:17. There are 80 dogs, in total, at this pet show. How many big dogs are there?
3. Karl had his salary reduced by 10%. He was later promoted and his salary was increased by 10%. If his original salary was \$ 20 000, what is his present salary?
4. Harry charges \$4 to babysit for the first hour. For each additional hour, he charges 50% more than he did for the previous hour. How much money in total would Harry earn for 4 hours of babysitting?
5. A fraction is equivalent to $\frac{5}{8}$. Its denominator and numerator add up to 91. What is the difference between the denominator and numerator of this fraction?
6. Naoki wrote nine tests, each out of 100. His average on these nine tests is 68%. If his lowest mark is omitted, what is his highest possible resulting average?
7. Lara ate $\frac{1}{4}$ of a pie and Ryan ate $\frac{3}{10}$ of the same pie. The next day Cassie ate $\frac{2}{3}$ of the pie that was left. What fraction of the original pie was not eaten?
8. Each time Kim pours water from a jug into a glass, exactly 10% of the water remaining in the jug is used. What is the minimum number of times that she must pour water into a glass so that less than half the water remains in the jug?
9. Enzo has fish in two aquariums. In one aquarium, the ratio of the number of guppies to the number of goldfish is 2:3. In the other, this ratio is 3:5. If Enzo has 20 guppies in total, what is the least number of goldfish that he could have?
10. Uncle Joe made chocolate chip cookies. Eric ate fifty percent of them right away. Ryan ate fifty percent of what was left. Ten cookies remain. How many cookies did Uncle Joe make?