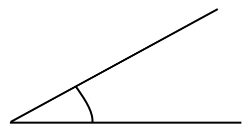
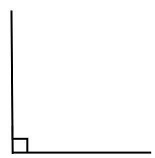
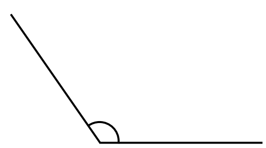
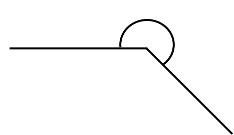
Geometry 2

1. Angle reviews

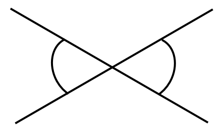
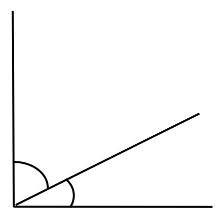
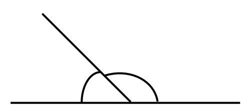
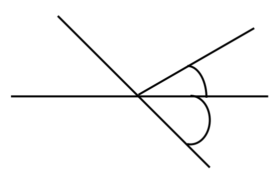
In this lesson, we will focus on angles; let's start with reviewing the basic angles types.

We can first classify angles according to their sizes.

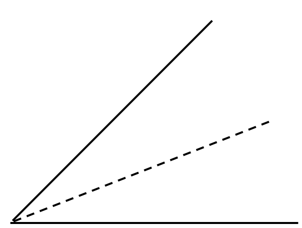
Angle type	*Example*	*Explanation*
Acute angle		The measurement of the angle is less than 90°; note that it does not include 90° itself.
Right angle		The angle measures exactly 90°.
Obtuse angle		The angle measures between 90° and 180°.
Straight angle		The angle is 180° which is also a straight line.
Reflex angle		It's between 180° and 360°.

Angle pairs

We can also name angle pairs according to their relationships.

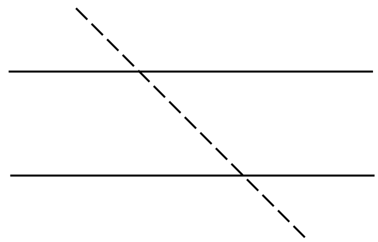
Angle type	*Example*	*Explanation*
Vertical angles (opposite angles)		The angles that are on the opposite side of each other when lines intersect each other; they have the same measurements.
Complementary angles		The angles' sum is 90°.
Supplementary angles		The angles' sum is 180°.
Adjacent angles		The angles that share a side; there is no specification of the angles' sizes.

Occasionally, students also see a concept named *angle bisector* (dashed line below), which divides an angle into equal halves.

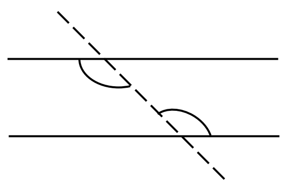
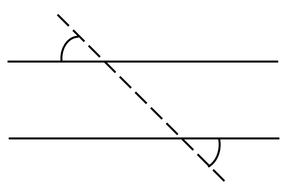
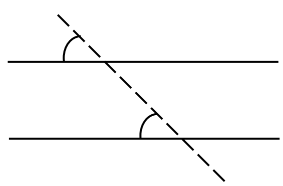
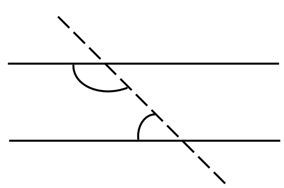


2. Angles and parallel lines

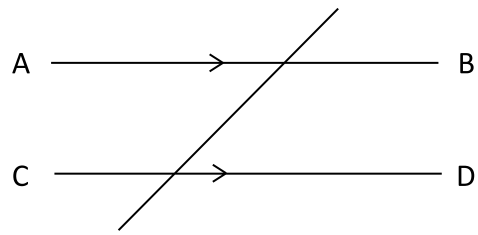
When a traversal intersects with a pair of parallel lines, it creates many sets of angles with special properties. In the below example, the dashed line is the traversal.



In terms of how the angle pairs are named, when the word "*interior*" is mentioned, it indicates that the angles are in between the parallel lines, or in another word, they are "inside" the set of parallel lines; similarly, "*exterior*" angles are those outside the pair of parallel lines.

Angle type	*Example*	*Explanation*
Alternative interior angles		They are located within the set of parallel lines and on the opposite sides of the traversal line. The angles are congruent.
Alternative exterior angles		They are located outside the set of parallel lines and on the opposite sides of the traversal line. The angles are congruent.
Corresponding angles		One angle is inside the parallel lines and one angle is outside; they are on the same side of the traversal line and they are congruent.
Co-interior angles		Both angles are within the set of parallel lines and they are on the same side of the traversal line. The angles are supplementary.

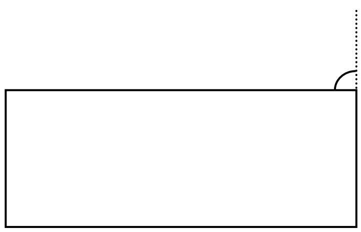
Note that besides written description, parallel lines can also be indicated by symbol "//" or arrows on the diagram, such as "$AB//CD$" or arrows in the below diagram.



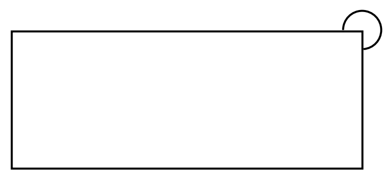
3. Exterior angles

An exterior angle is form between the side of a polygon and the adjacent side's extended line.

Example of an exterior angle:



Common misconception:

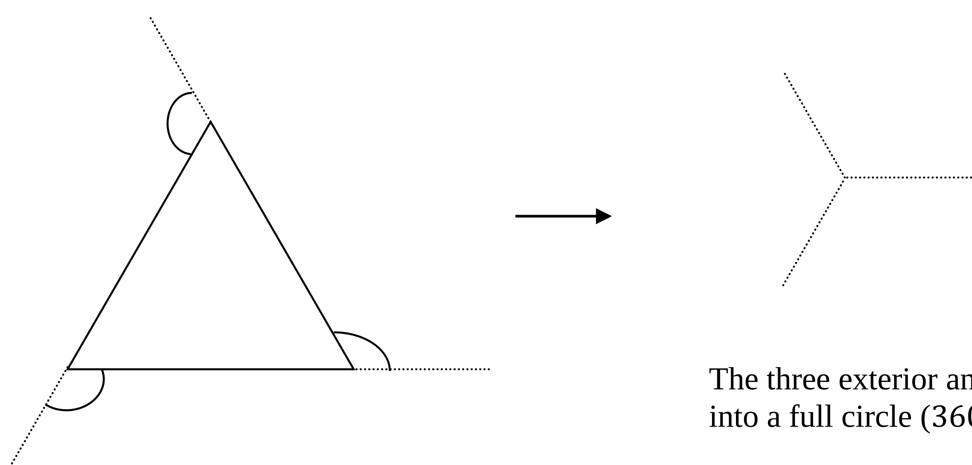


Unlike the sum of polygon's interior formula $(n-2)180°$, **the sum of the exterior angles is always $360°$**, which means an exterior angle in a regular polygon can be calculated with the below simple formula:

$$\frac{360°}{n}$$

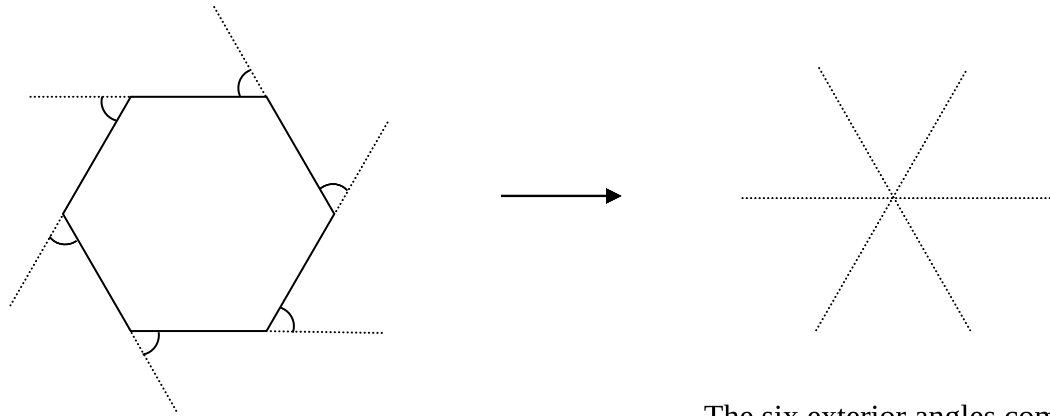
Let's explore this equation with two examples.

Example 1: equilateral triangle.



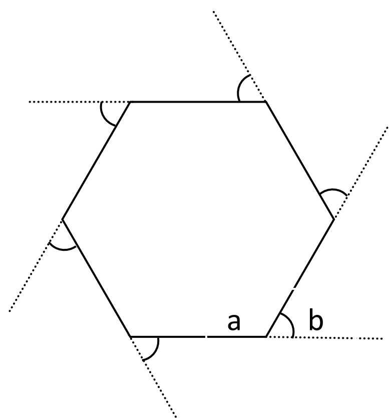
The three exterior angles combine into a full circle (360°)

Example 2: regular hexagon.



The six exterior angles combine into a full circle (360°).

Let's prove the formula with another example.



With formula $\frac{360°}{n}$, we can calculate the measurement of angle b as follows:

$$= \frac{360°}{6}$$

$$= 60°$$

Alternatively, we can calculate the measurement of the interior angle a first, and find the measurement of angle b by subtracting a's measurement from 180°, since they are supplementary angles.

Angle $a = \frac{(6-2)180°}{6}$

$= 120°$

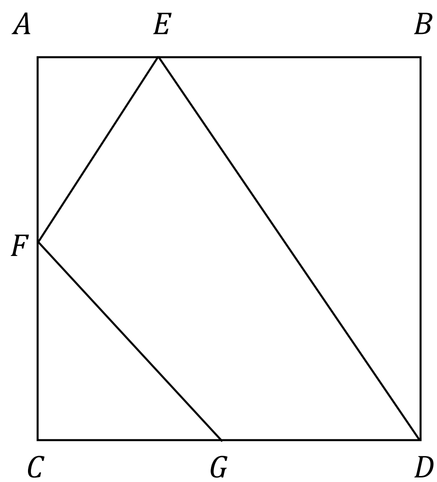
Angle $b = 180° - 120°$

$= 60°$

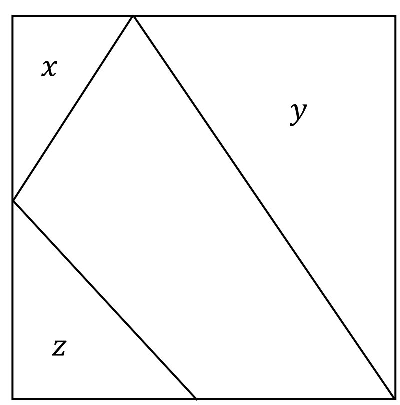
4. Calculate the area of unfamiliar shapes

We will present a technique for calculating the area of shapes that are unfamiliar to students, typically for those questions where the desired shape is inscribed within a square.

Discussion question: Square $ABCD$ has an area of 36; E trisects AB; F and G are midpoints. What is the area of quadrilateral $EFGD$?



Instead of calculating the area of quadrilateral $EFGD$, we can turn our attention to the three triangles that we would like to remove. We will name them x, y, and z.



Find the combined fraction that represents the areas of x, y, and z:

$$\frac{1}{3} \times \frac{1}{2} \div 2 + \frac{2}{3} \times 1 \div 2 + \frac{1}{2} \times \frac{1}{2} \div 2$$

$$= \frac{1}{12} + \frac{1}{3} + \frac{1}{8}$$

$$= \frac{2}{24} + \frac{8}{24} + \frac{3}{24}$$

$= \frac{13}{24}$

Area of quadrilateral $EFGD$:

$36 \times (1 - \frac{13}{24})$

$= 36 \times \frac{11}{24}$

$= 16\frac{1}{2}$

$$\therefore The\ area\ of\ \text{quadrilateral}\ EFGD\ is\ 16\frac{1}{2}.$$