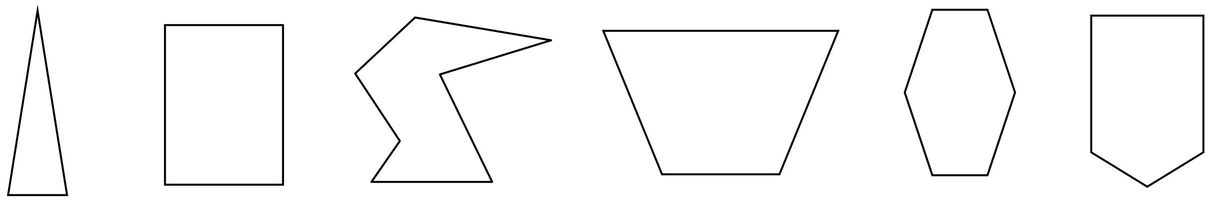
Geometry 1

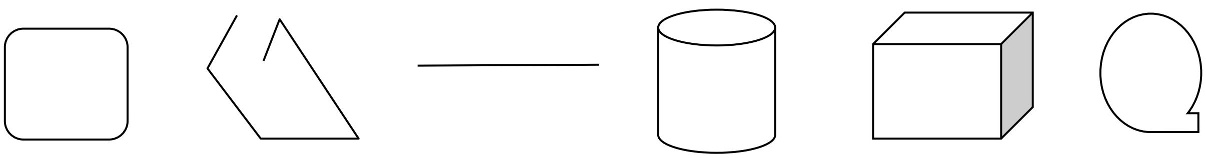
1. Polygons

Polygons are two-dimensional closed shapes made of straight lines.

Examples of polygons:



Examples that are not polygons:



The common reasons that disqualify figures from being polygons are as follows:

1. Part of the edge is not straight, as in the first and last figure about.

2. The shaped is not closed, as illustrated in the second example above.

3. The figure is not two-dimensional, such as the straight line, cylinder, and the rectangular prism in the examples.

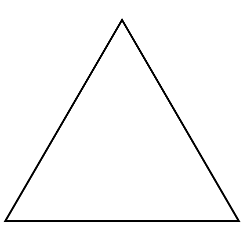
Fun fact: the reason we call a circle's total edge length *circumference* instead of perimeter is because circles are not polygons, since they have curved edges.

Within polygons, we can further classify them according to a number of attributes.

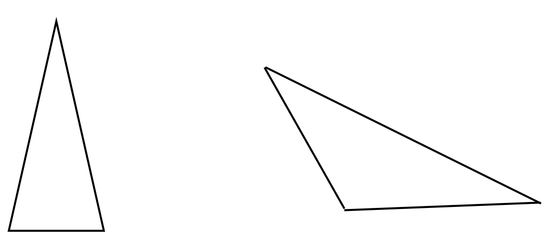
Regular vs. Irregular polygons

Regular polygons have congruent sides and angles, and irregular polygons don't.

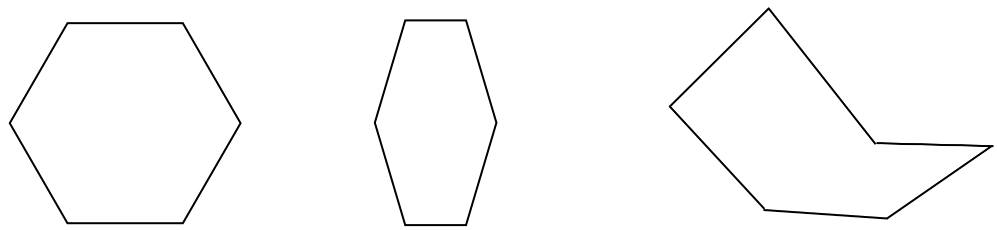
Regular triangle



Irregular triangles

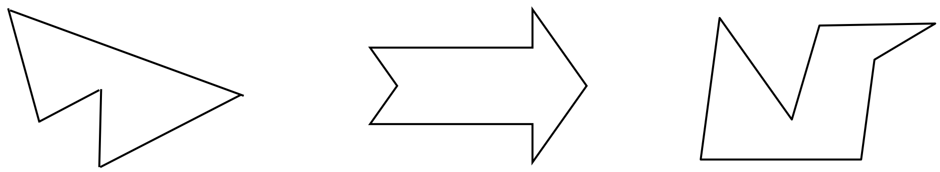


Regular hexagon Irregular hexagons



Convex vs. Concave polygons

Most of the polygons we have encountered so far are *convex* polygons, and below are examples that belong to the other category – *concave* polygons.



Did you notice the difference between convex and concave polygons?

At least one of the angles in concave polygons "cave in" or point inwards; on the other hand, convex polygons' angles all points outward instead.

Formally speaking, in concave polygons, at least one of the interior angles is greater than 180 degrees.

Polygon naming

In the context of polygons, variable n commonly represents the number of sides or angles.

$\boldsymbol{n}$	**Name**
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

One might ask, what about polygons with 1 or 2 sides? The minimum number of sides needed to form a closed 2D shape is 3; therefore, our chart starts with n equals to 3.

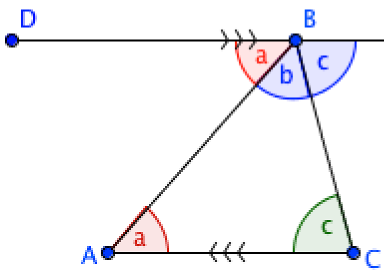
2. Sum of interior angles in polygons

Triangle sum theorem

The sum of all three interior angles in any triangle is always 180°, regardless the shape or the size of the triangles.

Let's derive this theorem with a diagram.

In the diagram below, $DE//AC$ (DE is parallel to AC).

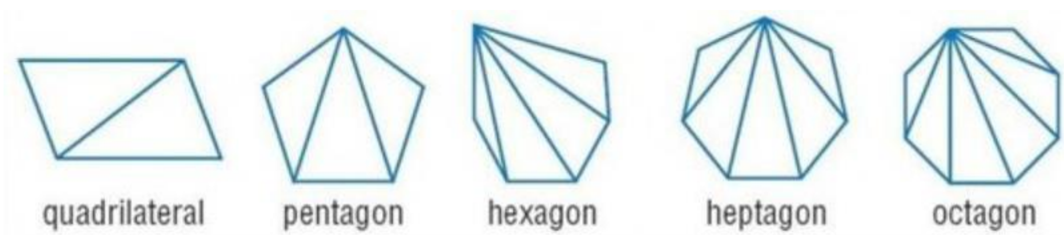


The pair of angles a are congruent and the pair of angles c are congruent as well, because they are alternative interior angles. Angles a, b, and c add up to 180° since they are on a straight line.

Angles a, b, and c are the three interior angles of the triangle in the above diagram, and we can deduce that the sum of interior angles inside a triangle is always 180°.

How about other polygons?

We can find the sum of the interior angles of any polygon so long as we know how many sides or angles this polygon has. To illustrate how this is achieved, we can first divide polygons into triangles.



Number of sides (n)	4	5	6	7	8
Number of triangles	2	3	4	5	6

Did you find a pattern between n and the number of triangles the polygon has?

The number of triangles in a polygon: $n - 2$

The sum of angles of a triangle is $180°$, and the sum of all angles inside of a polygon is therefore: $(n-2)180°$.

If the given polygon is irregular, we cannot find the measurement of its individual angles without further information, and knowing the sum of all interior angles is the best we can do.

However, if the given polygon is regular, we can certainly find the measurement of an individual angle since all of them are congruent.

Example 1: find the sum of angles and the measurement of an angle in a hexagon.

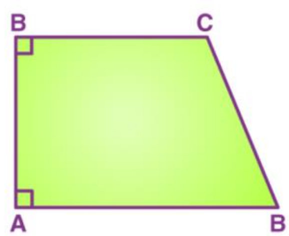
$n = 6$

$(6-2)180° = 720°$

$720° \div 6 = 120°$

$\therefore \textit{The sum of all angles is } 720° \textit{ and a single angle measures } 120°.$

Example 2: find the measurement of angle C; angle B measures $78°$.



The trapezoid is a quadrilateral and it has four sides, so its sum of angles is:

$(4-2)180° = 360°$

Three out of the four angles' measurements are known, and angle C can be obtained by:

$360° - 90° - 90° - 78° = 102°$

Example 3: Find the measurement of an individual interior angle in a regular heptagon.

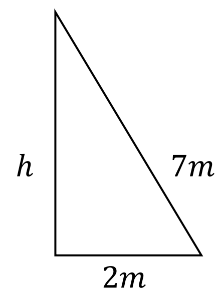
$\frac{(7-2)180}{7} = 128\frac{4}{7}$

In the case where the angle's measurement is not an integer, we can leave the answer in its fractional form or as rounded decimal.

3. Pythagorean theorem

Students are very familiar with Pythagorean theorem at this stage of learning, and the only challenge they might face with Pythagorean theorem is often related to the simplification. We will review this concept with an example.

Example: a 7 m-long-ladder's foot is 2 meters away from the wall, how tall is the point where the ladder touches the wall?



$$
\begin{aligned}
h &= \sqrt{7^2 - 2^2} \\
&= \sqrt{45} \\
&= \sqrt{9 \times 5} \\
&= \sqrt{9} \times \sqrt{5} \\
&= 3\sqrt{5}
\end{aligned}
$$

$$\therefore \textit{The height where the ladder touches the wall is } 3\sqrt{5}m.$$

When we simplify a square root number, its most reduced form is when the number under the square root sign does not have any square number factor (other than 1).

Examples of simplified numbers	Examples of not-yet-simplified numbers
$2\sqrt{2}$, $15\sqrt{19}$, $\sqrt{6}$, $\sqrt{77}$, $100\sqrt{5}$	$\sqrt{44}$, $\sqrt{450}$, $5\sqrt{60}$, $3\sqrt{128}$, $\sqrt{90}$, $\sqrt{300}$

If you attempted to simplify the numbers from above chart, here are the answers:

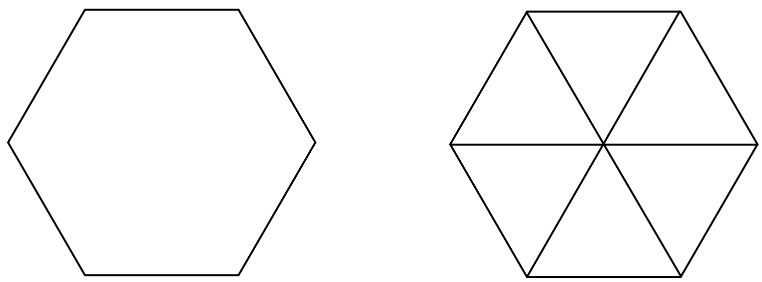
$\sqrt{44} = 2\sqrt{11}$ $\quad$ $3\sqrt{128} = 24\sqrt{2}$

$\sqrt{450} = 15\sqrt{2}$ $\quad$ $\sqrt{90} = 3\sqrt{10}$

$5\sqrt{60} = 10\sqrt{15}$ $\quad$ $\sqrt{300} = 10\sqrt{3}$

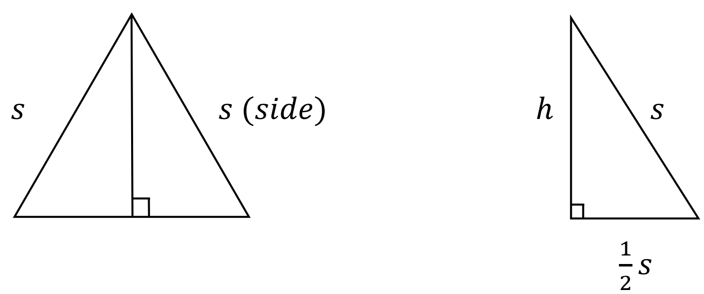
4. Regular hexagon

Although regular hexagon's area formula is not included in the grade 7 curriculum, it is still helpful to solving some challenging contest problems. Additionally, we will present the derivation process behind the regular hexagon area formula, and hopefully students can get inspired to derive the formula for unfamiliar shapes when needed.



We first divide the regular hexagon into more familiar shapes: six congruent equilateral triangles!

Let's focus on one of these triangles (the triangle is enlarged to make it more visible):



With Pythagorean theorem, we can find the height of the equilateral triangles.

$$h = \sqrt{s^2 - (\frac{1}{2}s)^2}$$

$$h = \sqrt{\frac{3}{4}s^2}$$

$$h = \frac{\sqrt{3}}{2}s$$

The hexagon's area is comprised of six such triangles, hence the formula:

$$6\,\frac{s\cdot(\frac{\sqrt{3}}{2}s)}{2}$$

$$= \frac{3\sqrt{3}}{2}s^2$$