# Word Problems 2

We will continue to discuss some interesting word problems throughout this chapter's note section.

***Question 1**: A can of soup can feed 3 adults or 5 children. If there are 5 cans of soup and 15 children are fed, how many adults would the remaining soup feed?*

Method 1: Algebra.

Every can of soup feeds 3 adults, so each adult consumes $\frac{1}{3}$ of a can.

Every can of soup feeds 5 children, so each child consumes $\frac{1}{5}$ of a can.

$$Let\ the\ number\ of\ adults\ the\ remaining\ soup\ feeds\ be\ x.$$

$$\frac{1}{3}x + \frac{1}{5} \cdot 15 = 5$$

$$\frac{1}{3}x + 3 = 5$$

$$x = 6$$

$$\therefore The\ remaining\ soup\ can\ feed\ 6\ adults.$$

Method 2: We can also solve this question without algebra.

Since the LCM of 3 and 5 is 15, we can divide a can of soup into 15 equal proportions.

Every can of soup feeds 3 adults, so each adult consumes 5 portions of a can.

Every can of soup feeds 5 children, so each child consumes 3 portions of a can.

In 5 cans, there are 75 portions of soup, and among which 45 portions have been consumed by children already, leaving 30 portions for adults, which is enough for 6 adults. See below calculation.

$$15\ portions/can \times 5\ cans = 75\ portions$$

$$5\ portions/child\ \times 15\ children = 45\ portions$$

$$75 - 45 = 30\ portions\ left$$

$$30\ portions\ \div 5\ portions/adult\ = 6\ adults$$

$$\therefore The\ remaining\ soup\ can\ feed\ 6\ adults.$$

***Question 2**: Kiki the farmer grows cantaloupes for a living, and most of her cantaloupes are 20 cm wide and 1.5 kg heavy. One day, she heard the cantaloupe that won this year's competition is 4 times as wide as hers, she wonders how much the king cantaloupe weights. Can you help her calculate it? You can assume the weight of the cantaloupe is proportion to its size.*

Method 1: Calculate the ratio of the volumes.

We can assume cantaloupes take the shapes of spheres, and the volume formula for a sphere is $\frac{4}{3}\pi r^3$ and Kiki's cantaloupe's radius is 10 cm.

Kiki's cantaloupe volume:

$$\frac{4}{3}\pi\ 10^3$$

$$= \frac{4}{3}\pi\ 1000\ cm^3$$

The king cantaloupe's volume:

$$radius\!:\ 10 \cdot 4 = 40\ cm$$

$$\frac{4}{3}\pi\ 40^3$$

$$= \frac{4}{3}\pi\ 64000\ cm^3$$

The king cantaloupe's weight:

$$\left(\frac{4}{3}\pi\ 64000\right) \div \left(\frac{4}{3}\pi\ 1000\right) = 64$$

$$1.5 \times 64 = 96\ kg$$

$$\therefore The\ king\ cantaloupe's\ weight\ is\ around\ 96\ kg.$$

Method 2: Scale factor. Scale factor is the change made to a single dimension of a shape.

$$New\ volume = Old\ volume\ \times Scale\ factor^3$$

The scale factor of changing from the Kiki's regular cantaloupe to the king cantaloupe is 4, since the questions mentioned that the king cantaloupe's width (diameter) is 4 times as much. We treat volume as a proxy to weight in this case.

$$New\ weight = Old\ weight \times Scale\ factor^3$$

$$= 1.5 \times 4^3$$

$$= 1.5 \times 64$$

$$= 96\ kg$$

$$\therefore The\ cantaloupe\ king's\ weight\ is\ around\ 96\ kg.$$

***Question 3**: Which of the following is the hypotenuse of a right triangle with leg lengths of 20 and 10 cm?*

A) $\sqrt{400}$ B) $\sqrt{400} + \sqrt{100}$ C) $10\sqrt{5}$ D) 25 E) 23

$$20^2 + 10^2 = hypotenuse^2$$

$$hypotenuse = \sqrt{500}$$

Notice that the answer $\sqrt{500}$ is nowhere to be found among the five choices, and the reason is that $\sqrt{500}$ is not at its most reduced format.

$$\sqrt{500}$$

$$= \sqrt{100 \times 5}$$

$$= \sqrt{100} \times \sqrt{5}$$

$$= 10\sqrt{5}$$

The answer is therefore choice C.

***Question 4**: How many real values can $n$ have in the following equation?*

$$(n-7)^2 = 25$$

Discussion: If we are to solve this equation, most of us will get 12 as the value of $n$, but $n$ has another value as well.

$$n - 7 = \pm 5$$

$$n = 5 + 7 \qquad n = -5 + 7$$

$$n = 12 \qquad n = 2$$

$$\therefore The\ variable\ n\ can\ have\ two\ real\ values.$$

***Question 5**: The sum of two natural numbers' squares is 300 and their product is 50; what is their sum?*

Discussion: We can better visualize this question by translating it into algebra equations.

$$Let\ x\ and\ y\ be\ the\ two\ numbers.$$

$$x^2 + y^2 = 300$$

$$xy = 50$$

$$x + y = ?$$

Although most students in our current course have not learned polynomials yet, we still recommend everyone to familiarize themselves with the common factorizations such as the following:

$$(x + y)^2 = x^2 + 2xy + y^2$$

It is a common misconception that $(x + y)^2$ should equal to $x^2 + y^2$, and let's see an example to dispel this myth.

If we are to plug in 2 and 3 into $x$ and $y$:

$$(x + y)^2 = 25$$

$$x^2 + y^2 = 13$$

We can see that the value of the two expressions do not equate; let's substitute the same values into $x^2 + 2xy + y^2$.

$$x^2 + 2xy + y^2 = 25$$

Let's now turn our attention back to the question and substitute the values into their corresponding expressions:

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x + y)^2 = (x^2 + y^2) + 2(xy)$$

$$(x + y)^2 = 300 + 2(50)$$

$$(x + y)^2 = 400$$

$$x + y = 20$$

$$\therefore The\ sum\ of\ the\ two\ numbers\ is\ 20.$$

***Question 6**: How many zeros appear at the end of 50 factorial?*

Discussion: $50!$ is equivalent to $50 \times 49 \times 48 \times 47 \times ... \times 3 \times 2 \times 1$; among the 50 numbers, we can get zeros in the end of the product from the following types of multiplication:

$$\_0 \times integer = \_0$$

$$\_5 \ \times even\ number = \_0$$

Since we have plenty of integers and even numbers, the limitation is the number of numbers that end in zero and five, which are both multiples of 5.

$$50 \div 5 = 10$$

There are ten numbers that can generate zeros at the end of the $50!$, which makes the number of zeros at least ten in the end of the 50!.

However, special numbers 25 and 50 will each generate an additional zero in the product:

$$25 \times 4 = 100$$
$$50 \times 2 = 100$$

$$10\ zeros + 2\ additional\ zeros = 12\ zeros$$

The answer is 12 zeros. We would like to elaborate on the reasons that 25 and 50 are special numbers.

The reasons that numbers ending in 0 and 5 generate zeros in the end of a product is that they contain 5 as a factor. In 25 and 50, there are two copies of factor 5:

$$25 = 5 \times 5$$
$$50 = 2 \times 5 \times 5$$

***Question 7**: How many seating arrangements are there for 8 students to be in a row if Cameron and Velda cannot sit together?*

Discussion: We can try to think of the opposite scenarios, where Cameron and Velda do sit together, and subtract the number of such arrangements from the total number of arrangements without any restrictions.

The total number of arrangements without any constraints:

$$8! = 40320$$

To calculate the number of scenarios where Cameron and Velda do sit together, we can “bundle” them up and treat them as one entity, which leave us with 7 entities in the group to be shuffled around. Additionally, there are 2! ways for Cameron and Velda to rearrange within the two of them:

$$7! \cdot 2! = 10080$$

The last step is to subtract the number of undesired scenarios from the total number of unrestricted scenarios:

$$40320 - 10080 = 30240$$

$$\therefore \mathit{There\ are\ } 30240 \mathit{\ arranegemts\ where\ Cameron\ and\ Velda\ do\ not\ sit\ together}.$$

***Question 8****: Which of the following is the closest to the value of* $2^{80}$*?*

A) $10^6$ B) $10^{10}$ C) $10^{15}$ D) $10^{20}$ E) $10^{24}$

Discussion: We are not to actually calculate the value of $2^{80}$, and instead, we will calculate an approximation of its value in base 10.

$$\begin{aligned} &2^{10} \\ &= 1024 \\ &\approx 10^3 \end{aligned}$$

Students might wonder, rightfully, can we round so much value off of 1024? We can do this only because the five choices in this question are multiple magnitudes apart, which affords us the luxury to round off a great amount of value without losing the right answer.

$$\begin{aligned} &2^{80} \\ &= (2^{10})^8 \\ &\approx (10^3)^8 \\ &= 10^{24} \end{aligned}$$

The correct choice is therefore E for this question.