Word problem 1

In this chapter's notes, we will explore some word problems and a number of techniques that students can utilize to solve these challenging questions.

Question 1*: Jojo the zoo keeper is responsible for taking care of penguins and the sea otters; today, she noticed that there are 22 heads and 66 feet in the animal enclosure. How many sea otters are in there?*

Method 1: Algebra.

We can make the number of sea otters be our variable x and represent all other values in terms of x as summarized in the table below. Note that penguins have two feet and sea otters have 4 feet each.

Animal	Number of heads	Number of feet
Sea otter	x	$4x$
Penguin	$22 - x$	$2(22 - x)$
	Total number of feet	$4x + 2(22 - x)$

From the table, we can represent the total number of feet as $4x + 2(22 - x)$, and the question also mentioned that the total number of feet is 66, hence the equation

$$\begin{aligned} 4x + 2(22 - x) &= 66 \\ 4x + 44 - 2x &= 66 \\ 2x &= 22 \\ x &= 11 \end{aligned}$$

$$\therefore There\ are\ 11\ penguins.$$

Method 2: Let's explore another technique to solve this question that does not use algebra.

If we assume all of the animals are penguins, which has 2 feet per individual, there would be 44 feet. However, there are 22 "extra" feet, which must belong to the sea otters. Every sea otter has 2 "extra" feet compared to penguin, and with a total of 22 "extra" feet, there must be 11 sea otters. See below calculations.

$$\begin{gathered} 2 \times 22 = 44 \\ 66 - 44 = 22 \\ 4 - 2 = 2 \\ 22 \div 2 = 11 \end{gathered}$$

$$\therefore There\ are\ 11\ penguins.$$

Question 2*: The prices for mini horses and pygmy goats are $560 and $320; the animal vendor counted all his mini horses and pygmy goats and realized that he could make $13840 if he sells all of them at the upcoming pet convention. How many pygmy goats are there?*

Discussion: Although there are also two types of animals in this question, we need to approach it differently because we were not provided with the total number of animals like in the previous example.

One interesting fact we can utilize is that the head counts for the animals in this question must be whole numbers. We can make one horse and one goat a “bundle”, and calculate how many such bundles are there in the $13840 total amount.

$$13840 \div (560 + 320) = 15\ R\ 640$$

It seems that with 13840, we can purchase 15 bundles of animals, which are 15 horses and 15 goats, along with an additional $640 worth of animals. Given the prices of the animals, $640 must be from the goats because it is not divisible by 560, the horse’s price. How many goats are worth $640?

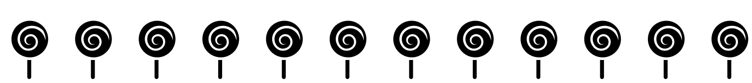
$$640 \div 320 = 2$$
$$15 + 2 = 17$$

$$\therefore There\ are\ 17\ pygmy\ goats.$$

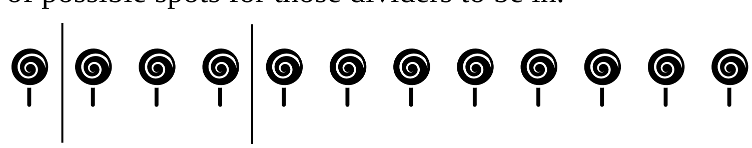
Question 3*: Cameron found a pile of 15 lollipops on the ground and he decided to share it with two of his friends; he is a generous boy and wants everybody to have at least 2 lollipops. How many ways are there for them to split the lollipops?*

Discussion: With 15 lollipops, it is not so feasible to list all of the combinations and we need to find a faster way to solve the problem.

If we are to give everyone one lollipop to start with, there will be 12 lollipops left to split among three people, and they each receive 1 or more lollipops. See the diagram below.



Instead of focusing on separating the 12 lollipops into 3 groups, we can take two dividers and consider the number of possible spots for those dividers to be in.



There are 11 potential spots for the first divider, and the second divider will have only 10 choices of spot, since they cannot overlap. With two dividers, the lollipops are divided into three groups.

$$11 \times 10 = 110$$

However, the order of the dividers does not matter here which means there are duplicates in the number of combinations.

$$110 \div 2! = 55$$

$$\therefore \textit{There are } 55 \textit{ ways to split the lollipops.}$$

Question 4*: What is the probability of rolling two standard dice and the product of the outcome is an even number?*

Discussion: There are three scenarios where we can receive an even product.

$$even \times even = even$$

$$even \times odd = even$$

$$odd \times even = even$$

However, there is only one scenario to receive an odd product.

$$odd \times odd = odd$$

Instead of computing the probability for the three even product scenarios, we can subtract the chance of receiving odd products from 1 to solve the question.

$$P(even\ product) = 1 - P(odd\ product)$$

$$P(odd\ product) = P(1st\ result\ is\ odd) \times P(2nd\ result\ is\ odd)$$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{4}$$

$$P(even\ product) = 1 - P(odd\ product)$$

$$= 1 - \frac{1}{4}$$

$$= \frac{3}{4}$$

$$\therefore \textit{The probability of receiving an even product is } \frac{3}{4}.$$

Question 5*: Emma spent $234 to buy 19 chicken and 12 ducklings; she realized that if she reversed the number of chickens and ducklings she bought, it would have costed $262 instead. How much would it cost to purchase a pair of chickens and a pair of ducklings?*

Discussion: We can use algebra to solve this question by making two variables, and we can then make two equations to represent the costs of these animals. Note that we can simply swap the position of the two variables to represent "reverse the number of chickens and ducklings".

$$Let\ c\ and\ d\ be\ the\ price\ of\ a\ chicken\ and\ a\ duckling,\ respectively.$$

$$19c + 12d = 234$$
$$19d + 12c = 262$$

Method 1: Solve for the unit price.

$$19c + 12d = 234$$

$$c = \frac{234 - 12d}{19}$$

Substitute the c in the second equation with the right-hand side expression from the above equation.

$$19d + 12\frac{234-12d}{19} = 262$$

$$361d + 12(234 - 12d) = 4978$$

$$361d + 2808 - 144d = 4978$$

$$217d = 2170$$

$$d = 10$$

Substitute the value of d into a previous equation to compute the value of c.

$$c = \frac{234-12d}{19}$$

$$c = \frac{234-12(10)}{19}$$

$$c = 6$$

The price for two chickens and two ducklings:

$$2 \times 10 + 2 \times 6 = 32$$

$$\therefore The\ price\ for\ a\ pair\ of\ chickens\ and\ a\ pair\ of\ ducklings\ is\ \$32.$$

Method 2: Without finding the unit prices.

First, add the two equations together.

$$\begin{array}{r} 19c + 12d = 234 \\ 19d + 12c = 262 \\ \hline 31c + 31d = 496 \end{array}$$

Divide both sides of the equation by 31.

$$c + d = 16$$

Multiply both sides of the equation by 2 to find out the price for 2 chickens and 2 ducklings.

$$2(c + d) = 32$$

$$\therefore The\ price\ for\ a\ pair\ of\ chickens\ and\ a\ pair\ of\ ducklings\ is\ \$32.$$

Question 6*: When Socrates' dad drove him to school on Friday, it took 20 minutes. However, the trip home was 18km/h faster and used only 12 minutes because the snow has been removed. How far is the school?*

Caution: the times are in minutes and the speed is in kilometer per *hour*. Let's convert the time into hours first.

$20\ minutes = \frac{1}{3}\ hour$ $\qquad$ $12\ minutes = \frac{1}{5}\ hour$

The relationship we are using for making the equation:

$$speed\ for\ the\ first\ trip + 18 = speed\ for\ the\ return\ trip$$

$$Let\ the\ distance\ for\ a\ single\ trip\ be\ d.$$

$$\frac{d}{\frac{1}{3}} + 18 = \frac{d}{\frac{1}{5}}$$

$$3d + 18 = 5d$$

$$18 = 2d$$

$$d = 9$$

$$\therefore The\ distance\ to\ school\ is\ 9km.$$

***Question 7**: Which of the following number cannot be written as the sum of four consecutive odd integers?*

A) 16 B) 40 C) 72 D) 100 E) 200

Discussion: We can make a variable and try to find the patterns of the sum of four consecutive odd integers.

$$Let\ the\ first\ odd\ number\ be\ x.$$

The sum of four consecutive odd numbers can be represented with the following expression:

$$x + (x+2) + (x+4) + (x+6)$$

$$= 4x + 12$$

The result suggests that the sum of four consecutive odd integers should be a multiple of 4 plus 12.

However, among the five choices, it seems that all of them can be expressed as a multiple of 4 plus 12:

A) $16 = 1 \times 4 + 12$ $\quad x = 1$

B) $40 = 7 \times 4 + 12$ $\quad x = 7$

C) $72 = 15 \times 4 + 12$ $\quad x = 15$

D) $100 = 22 \times 4 + 12$ $\quad x = 22$

E) $200 = 47 \times 4 + 12$ $\quad x = 47$

If we are to read our “let” statement again, notice that x must be an odd inter. Among the five choices, x equals to 22 for choice D), which makes it the answer since 22 is an even number.