Algebra

Algebra is the branch of mathematics that uses symbols (usually letters) rather than numbers in arithmetic.

There are very specific math format rules that we need to follow when using algebra; we recommend all new students to read the below rules in detail, and the return students can use them as a refresher.

1. Math format rules for basic calculation

Before diving into solving algebra equations, we need to first learn the correct way to work with algebra.

a) Sign omission

To many students' surprises, we do not use " × " or " ÷ " signs in this chapter. See below chart for the alternatives to represent multiplication and division.

Wrong Format	$6 \times x$	$12 \times 9 - 3x = 3$	$b \div 5$	$8 + 99 \div c = 15$
Right Format	$6x$ $6 \cdot x$	$12 \cdot 9 - 3x = 3$ $12(9) - 3x = 3$	$\frac{b}{5}$	$8 + \frac{99}{c} = 15$

b) Show variable(s) in every step

The variable needs to be present in every step; it can be on the left or the right side.

Wrong format	Comments	Correction
$3b = 15$ $\frac{15}{3} = 5$ $b = 5$	b disappeared	$3b = 15$ $b = \frac{15}{3}$ $b = 5$

c) Using equal signs properly

There should only be one equal sign per row, and the equal sign should be in between 2 expressions.

Wrong format	Correction
$x - 50 = 37$ $x = 37 + 50 = 87$	$x - 50 = 37$ $x = 37 + 50$ $x = 87$
$4x = 79 - 15(3)$ $= 79 - 45$ $=34$ $4x = 34$ $x = \frac{34}{4}$	$4x = 79 - 15(3)$ $4x = 79 - 45$ $4x = 34$ $x = \frac{34}{4}$

d) Final step

In the final step, the variable should be on the left side of the equation.

Wrong format	Correction
$43 = 3j - 17$ $43 + 17 = 3j$ $60 = 3j$ $\frac{60}{3} = j$ $20 = j$	$43 = 3j - 17$ $43 + 17 = 3j$ $60 = 3j$ $\frac{60}{3} = j$ $j = 20$

2. Math format rules for word problems

Students are required to use algebra to solve all word problems in this unit's homework. You may wonder "why should I use algebra if I know how to solve it in other ways?"

It is true that many questions can be solved in more than one way; however, the purpose of this chapter is to sharpen student's fluency in algebra, which is essential to succeed in mathematics. Outside of the algebra chapters, students can use their preferred method, but we hope all students receive a good amount of algebra practice to develop this fundamental pillar of their math competency.

Example: Five more than 3 times a number is 26, what is the number?

Wrong solution 1:

$(26 - 5) \div 3 = 7$

Wrong solution 2:

$$x = \frac{(26-5)}{3}$$

$$= 7$$

The first example is wrong because algebra was not used.

The second example attempted to use algebra, but the variable x did not participate in the calculation at all, which makes it a "fake" algebra.

Whenever the variable is the only term on its side of the equation for the first step, it is not real algebra.

In order to make a proper algebra equation, students should first make a "let" statement. The variables we use such as a, b, c, x, y, z are like code names, which will remained unexplained unless we specific what do they represent, and a "let" statement serves this important function.

"Let" statement is mandatory whenever we solve a word problem (unless the question provided the variable already). The statement should be a complete English sentence without any symbols.

Wrong "let" statement	Right "let" statement
$x = the\ number$ $Let\ x = the\ number$ $Let\ x\ be\ the\ \#$	$Let\ x\ be\ the\ number$

Once the "let" statement is written, we can then translate the question into an algebra equation.

Five more than 3 times a number is 26, what is the number?

$$\underset{\downarrow}{5} \quad \underset{\downarrow}{+} \quad \underset{\downarrow}{3} \quad \underset{\downarrow}{\cdot} \quad \underset{\downarrow}{x} \quad \underset{\downarrow}{=} \; 26$$

The math format rules described on page one of this document apply to all algebra questions include the word problems. To sum it up, below the correct way to solve this question with the proper math format.

Five more than 3 times a number is 26, what is the number?

$Let\ x\ be\ the\ number$

$$5 + 3 \cdot x = 26$$

$$3x = 26 - 5$$

$$3x = 21$$

$$x = 7$$

$\therefore The\ number\ is\ 7.$

3. Using algebra expressions

Typically, when algebra is used for a question, students make an algebra *equation* with the and solve it. However, we can sometimes use algebra *expressions* instead to solve the question.

Equation	Expression
An equation is made of two expressions connected by an equal sign.	An expression is a number, a variable, or a combination of numbers and variables joined by operation symbols.
We can usually solve for the variables.	We are often not expected to solve for the variable's value because we do not know the value of the whole expression, but algebra expressions can still offer important insights in solving problems.
$3x + 10 = 99$	$3x + 10$

Let's see an example where algebra expressions are used to solve a word problem.

Example: Which of the following cannot be the sum of three consecutive natural numbers?

a) 36 b) 98 c) 144 d) 3000 e) 111111

Solution:

Let the first number be x.

$$\begin{aligned} & x + (x + 1) + (x + 2) \\ &= 3x + 3 \\ &= 3(x + 1) \end{aligned}$$

Although we cannot solve for the value of x, the sum of three consecutive natural numbers $3(x + 1)$ indicates that the result must be a multiple of three, since $x + 1$ is a natural number.

The answer is therefore b.

4. Technique for solving algebra word problems

In this section, we will discuss some techniques that will come handy when solving harder algebra questions.

a) Represent digits vs represent values

Sample question: I noticed that when I interchanged the digits of my father's age, I got my own age. When I was born, his age was between twenty and thirty years. What was my father's age when I was born?

$$Let\ the\ father's\ current\ age\ be\ xy, or\ 10x + y.$$

If we are to interchange the digits in the father's age, it will be yx, or $10y + x$.

Father's current age – My current age = Father's age when I was born.

$$(10x + y) - (10y + x)$$

$$= 10x + y - 10y - x$$

$$= 9x - 9y$$

$$= 9(x - y)$$

Although we cannot solve a stand-alone algebra expression, we do know additional information regarding x and y.

Since x and y are the digits of ages, they must be natural number, which means the father's age when the child was born, $9(x-y)$, is a multiple of nine. The question also mentioned that the father's age was between 20 and 30 at that time, like solving a puzzle, we realized that there is only one multiple of nine in that number range, which is 27.

b) Utilizing charts

Using charts can be particularly helpful to visualize the relationship among multiple algebra expressions involved in the questions; we will demonstrate this point with a classic example.

Sample question: Jensen has 20 animals and he counted 68 feet. He only has two kinds of animals: guinea pigs and silkie chicken. How many silkie chicken does he have?

$$Let\ the\ number\ of\ silkie\ chicken\ be\ x.$$

	Guinea pig	Silkie chicken
Head count	$20-x$	x
Feet count	$4(20-x)$	$2x$

Guinea pig feet + Silkie chicken feet = 68

$$4(20-x)+2x=68$$

$$80-4x+2x=68$$

$$80-2x=68$$

$$x=12$$

$$\therefore There\ are\ 12\ silkie\ chicken.$$