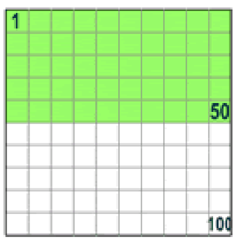
Percentages

1. Definition

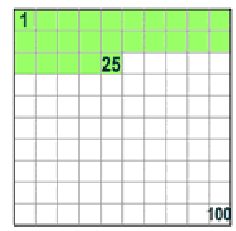
The word *Percent* comes from the Latin word *Per Centum*, in which the word *Centum* means 100; for example, a *century* has 100 years.

When we say "percent", we are really saying "per 100", or “out of 100”.



50% means 50 per 100
(50% of this box is green)

25% means 25 per 100
(25% of this box is green)



2. Conversions

Percent to decimal

Divide the number in the percentage by 100, or move the decimal point two places to the left and remove the percentage sign.

Decimal to percent

Multiply the decimal by 100, or move the decimal point two places to the right and add a percentage sign.

Percent to fraction

Step 1: Rewrite the percentage as a fraction out of 100.

Step 2: If the numerator is not a whole number, multiply both the numerator and denominator by powers of 10 (such as 10, 100, and etc.) until there is no decimal within the fraction.

Step 3: Simplify the fraction.

Example: Express 62.5% as a fraction.

Step 1:

$$62.5\% = \frac{62.5}{100}$$

Step 2:

$$\frac{62.5}{100} = \frac{625}{1000}$$

Step 3:

$$\frac{625}{1000} = \frac{5}{8}$$

Fraction to percent

The most common method to convert a fraction into a percent is to convert the fraction into a decimal first, and we can subsequently multiply the decimal by 100 and add a percentage sign to complete the conversion.

Example: Express $\frac{3}{16}$ as a percent.

$$3 \div 16 = 0.1875$$

$$0.1875 = 18.75\%$$

3. Basic word problems

Example 1: 15% of 200 apples had gone bad. How many apples are bad?

$$15\% = 0.15$$

$$0.15 \times 200 = 30$$

Example 2: If only 10 of the 200 apples were bad, what percent is that?

$$\frac{10}{200} = 0.05$$

$$0.05 \times 100\% = 5\%$$

Example 3: A Skateboard's price is reduced 25% for Black Friday sales. The old price was $120. Find the new price.

$25\% = 0.25$

$0.25 \times 120 = 30$

$120 - 30 = 90$

Or

$120 \times (1 - 25\%)$

$= 120 \times 75\%$

$= 90$

4. Percentage change

We hereby present two ways to calculate the percentage difference and students may use whichever method they prefer:

Method 1

Step 1: Calculate the difference of the two numbers.

Step 2: Divide that difference by the old value to receive a decimal.

Step 3: Convert the decimal into a percentage.

We can summarize this method into a formula:

$$\frac{New\,Value\ -\ Old\,Value}{Old\,Value} \times\ 100\%$$

Method 2

Step 1: Divide the new value by the old value to receive a decimal number.

Step 2: Convert the decimal into a percentage.

Step 3: Subtract 100% from the result from the previous step.

Example: The price of a pair of socks went from $5 to $6; what is the percentage difference?

Note: we recommend students to identify the old and new numbers first before the caculation. In this case, $5 is the old price and $6 is the new price.

Method 1:

$$(6-5) \div 5$$
$$= 0.2$$
$$= 20\%$$

Method 2:

$$6 \div 5 = 1.2$$
$$1.2 = 120\%$$
$$120\% - 100\% = 20\%$$

5. Common misconceptions

When we work with percentages in the context of word problems, keep in mind that the number that the percentage applies to can change. We will elaborate this concept with the following scenarios.

a) The number 100 is first increased by 20% and then by 30%, what is the final percentage change?

Wrong solution:

$$20\% + 30\% = 50\%$$

Proof:

$$100 \times (1 + 20\%) = 120$$

After a 20% increase, the number is 120.

$$120 \times (1 + 30\%) = 156$$

The 30% change is based off of the latest number 120, instead of the old number 100.

Correct solution:

$$(1 + 20\%) \times (1 + 30\%) - 100\%$$
$$= 1.2 \times 1.3 - 100\%$$
$$= 1.56 - 100\%$$
$$= 56\%$$

b) Toronto's population has been increasing 20% every year. The population is 3,600,000 this year, what was the population last year?

Wrong solution:

$$3{,}600{,}000 \times (1 - 20\%)$$

$$= 3{,}600{,}000 \times 0.8$$

$$= 2{,}880{,}000$$

Proof:

$$2{,}880{,}000 \times (1 + 20\%) = 3{,}456{,}000$$

The number does not match the 3,600,000.

Correct solution:

$$Old\ number \times (1 + 20\%) = New\ number$$

$$\begin{aligned} Old\ number &= New\ number \div (1 + 20\%) \\ &= 3{,}600{,}000 \div (1 + 20\%) \\ &= 3{,}000{,}000 \end{aligned}$$

c) Wilfred's salary was first increased by 20% then decreased by 20%. Wilfred thinks it's not a big deal since there is no change to his salary. Is he correct?

Wrong answer:

$$+20\% - 20\% = 0$$

Thus, there is no change.

Proof:

Suppose Wilfred's original salary was 5000.

$$5000 \times (1 + 20\%) = 6000$$

$6000 \times (1 - 20\%) = 4800$

He is making less than before!

Correct solution:

$(1 + 20\%) \times (1 - 20\%)$

$= 1.2 \times 0.8$

$= 0.96$

$100\% - 96\% = 4\%$

Wilfred actually lost 4% of his original salary.

d) Joyko's farm is rectangular shaped with a dimension of 200m by 100m. She plans to change the shape of her farm to better manage the animals. She wants to decrease the length by 20% and increase the width by 20% but she is not sure if she will have the same amount of land after the change. Can you help her?

Wrong answer:

One side's 20% decrease is cancelled out by the other side's 20% increase, so the area stays the same.

Proof:

$Length: 200 \times (1 - 20\%) = 160$ $\quad$ $Width: 100 \times (1 + 20\%) = 120$

$Old\ area: 200 \times 100 = 20000$ $\quad$ $New\ area: 160 \times 120 = 19200$

Correct solution:

$Length \times (1 - 20\%) \times Width \times (1 + 20\%)$

$= Length \times Width \times 0.8 \times 1.2$

$= Length \times Width \times 0.96$

$= Old\ area \times 0.96$

The new area is 0.96, or 96% of the old area; or in another word, Joyko would lose 4% of the land.